DTU Library

Multi-block, boundary-fitted solutions for 3D nonlinear wave-structure interaction

Bingham, Harry B.; Ducrozet, Guillaume; Engsig-Karup, Allan Peter

Published in:
International Workshop on Water Waves and Floating Bodies

Publication date:
2010

Document Version
Publisher's PDF, also known as Version of record

Multi-block, boundary-fitted solutions for 3D nonlinear wave-structure interaction *

Harry B. Bingham
Mech. Engineering
Tech. Univ. of Denmark
E-mail: hbb@mek.dtu.dk

Guillaume Ducrozet
Fluid Mechanics
Ecole Centrale de Nantes
Guillaume.Ducrozet@ec-nantes.fr

Allan P. Engsig-Karup
Informatics
Tech. Univ. of Denmark
apek@imm.dtu.dk

Introduction

This abstract describes the application of the high-order finite difference model OceanWave3D to the diffraction of nonlinear waves around a fixed, bottom mounted circular cylinder. We also discuss extension of the model from one structured geometric block to multiple overlapping blocks which will allow for the treatment of more general fixed and floating structures. This work builds on some preliminary results showing linear solutions on one curvilinear block which were presented at the 24th workshop [2]. The basic methodology is presented in detail in [1, 3], which includes stability and accuracy analyses in both two- and three-dimensions, together with a range of validation tests demonstrating the accuracy and the efficiency of the model on one block. The goal of this work is a computational tool suitable for large-scale prediction of nonlinear wave-wave, wave-bottom and wave-structure interaction in the coastal and offshore environment.

Formulation

A Cartesian coordinate system is adopted with the xy-plane located at the still water level and the z-axis pointing upwards. The still water depth is given by $h(\boldsymbol{x})$ with $\boldsymbol{x} = (x, y)$ the horizontal coordinate. The position of the free surface is defined by $z = \zeta(\boldsymbol{x}, t)$ and the gravitational acceleration $g = 9.81m^2/s$ is assumed to be constant.

Assuming a potential flow, the fluid velocity $(\boldsymbol{u}, w) = (u, v, w) = (\boldsymbol{\nabla}\phi, \partial_z\phi)$ where $\boldsymbol{\nabla} = (\partial_x, \partial_y)$ is the horizontal gradient operator. The evolution of the free surface is governed by the kinematic and dynamic boundary conditions

$$\partial_t\zeta = -\boldsymbol{\nabla}\zeta \cdot \boldsymbol{\nabla}\tilde{\phi} + \tilde{w}(1 + \boldsymbol{\nabla}\zeta \cdot \boldsymbol{\nabla}\zeta), \tag{1a}$$

$$\partial_t\tilde{\phi} = -g\zeta - \frac{1}{2}\left(\boldsymbol{\nabla}\tilde{\phi} \cdot \boldsymbol{\nabla}\tilde{\phi} - \tilde{w}^2(1 + \boldsymbol{\nabla}\zeta \cdot \boldsymbol{\nabla}\zeta)\right), \tag{1b}$$

expressed in terms of the free surface quantities $\tilde{\phi} = \phi(\boldsymbol{x}, \zeta, t)$ and $\tilde{w} = \partial_z\phi|_{z=\zeta}$. To find $\tilde{w}$ and evolve these equations forward in time requires solving the Laplace equation in the fluid volume with a known $\tilde{\phi}$ and ζ, together with the free-slip condition on all solid boundaries:

$$\phi = \tilde{\phi}, \quad z = \zeta, \tag{2a}$$

$$\boldsymbol{\nabla}^2\phi + \partial_{zz}\phi = 0, \quad -h \leq z < \zeta, \tag{2b}$$

$$(\boldsymbol{n}, n_z) \cdot (\boldsymbol{\nabla}; \partial_z)\phi = 0, \quad (\boldsymbol{x}, z) \in \partial\Omega \tag{2c}$$

where $(\boldsymbol{n}, n_z)$ is an outward pointing normal vector to the rigid boundary surface $\partial\Omega$.

*The authors wish to thank Délégation Générale de l'Armement (project PDE 08C0039) for partially funding this work, and the Danish Center for Scientific Computing for supercomputing resources.

An overlapping block description of the geometry

A 2D schematic example of the overlapping block discretization of the geometry is shown in Figure 1. This figure shows the interior of a three-block disretization of a circular

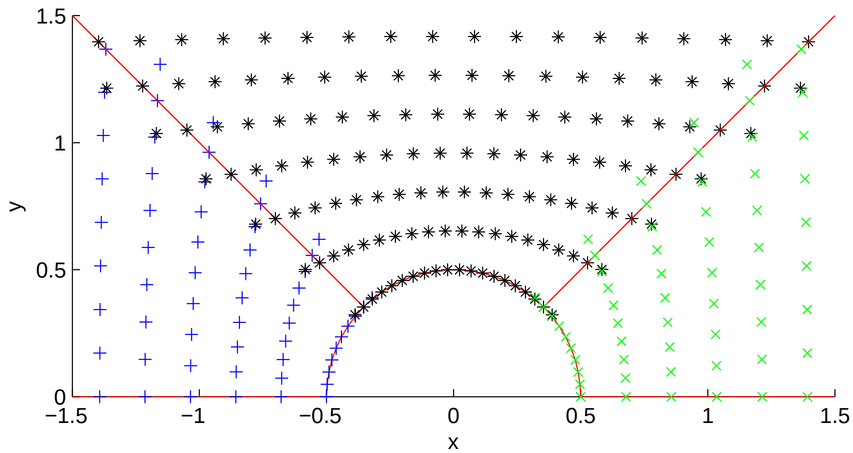

Figure 1: A three-block overlapping grid around a cylinder.

cylinder where the different symbols represent grid points on each of the blocks. Each block is structured, and has its own boundary fitted curvilinear coordinate system defined by the transformation between the physical grid point positions $[x(\xi,\eta), y(\xi,\eta)]$ and the computational unit-spaced square grid $[\xi(x,y), \eta(x,y)]$. Using the chain rule, all partial derivatives in a physical domain can be expressed in the computational space. For example, $\partial_x = (y_\eta/J)\partial_\xi + (y_\xi/J)\partial_\eta$ with $J = x_\xi y_\eta - x_\eta y_\xi$, gives the x-derivative evaluated entirely in the computational space where terms like $x_\xi = \partial x/\partial\xi$ are the grid transformation weights.

At a grid point which overlaps onto a neighboring block, continuity of the solution is imposed by equating the point value with that given by an interpolating polynomial passed through nearby points on the neighbor grid. This interpolation is done to an order which is consistent with the rest of the solution. These connecting equations are then combined with those arising from the Laplace problem on each block which is discretized as described in [3]. The resulting linear system of equations is solved using the preconditioned GMRES algorithm described in [3], although multigrid preconditioning has not yet been tested on curvilinear blocks.

Nonlinear decomposition of the solution

For wave-structure interaction problems, it is convenient to make a fully nonlinear decomposition of the solution into incident and scattered components. This idea has been applied in several previous publications *e.g.* [6] and is based on expressing the solution as

$$\zeta = \zeta^I + \zeta^S, \qquad \phi = \phi^I + \phi^S, \tag{3}$$

with ζ^I and ϕ^I given explicitly by some theory, (*e.g.* stream function theory [4]). Inserting (3) into the free-surface conditions (1) and isolating $\partial_t\zeta^S$ and $\partial_t\tilde{\phi}^S$ gives new evolution equations for the scattered component. Note however, that this procedure requires the evaluation of ϕ^I on the position $\zeta = \zeta^I + \zeta^S$ which can be above the incident wave free-surface

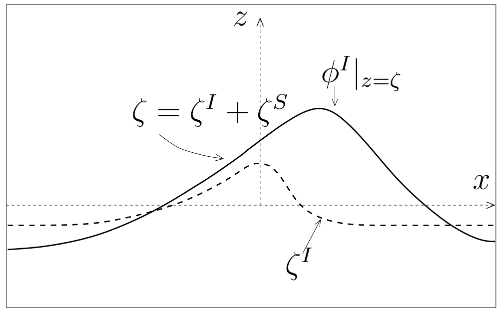

Figure 2: Nonlinear decomposition of the wave-structure interaction problem.

ζ^I as pictured in Figure 2. This means that whatever theory is used for the incident wave must allow for an analytic continuation of the solution above the free-surface. Fortunately this is generally the case for solutions arising from potential flow, and it turns out that the procedure is robust even for the steepest possible waves. An example is shown in Figure 3 which shows the reflection of a deep water nonlinear incident wave from a wall to produce a standing wave. Comparison is also made in this figure with the experimental measurements of [7] for the steepest stable deep water standing wave. The numerical calculations here were obtained by increasing the incident wave height until the solution began to go unstable.

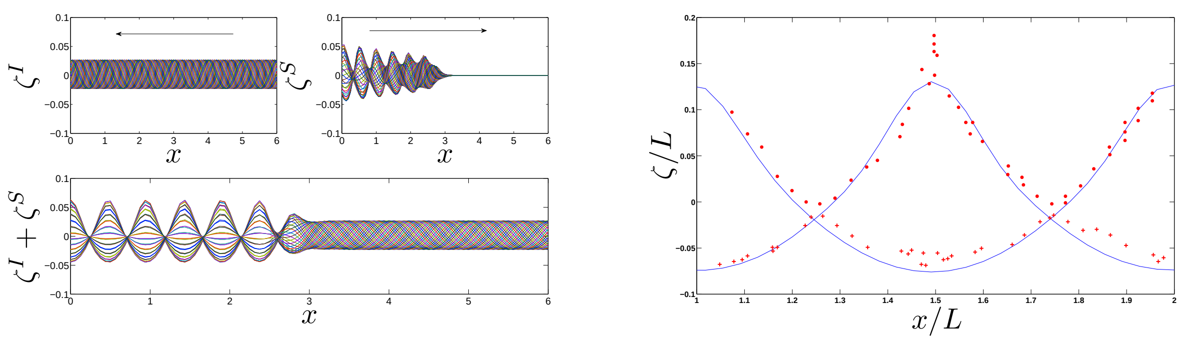

Figure 3: Nonlinear standing waves by reflection from a wall. Left: The two components and their sum. Right: Comparison with experiments by [7] for the steepest stable standing wave.

As a first 3D test case with a structure, we now consider the diffraction of waves from a bottom mounted circular cylinder. The linear problem was first validated using the known analytic solution, but due to lack of space those results are not included here. In Figure 4 we compare fully nonlinear calculations of the wave run-up around a cylinder with Stokes 2nd-order theory and measurements made by [5] for four of the cases given in that paper. The cases shown here are at $kh = 1.036$, and $kR = 0.374$ (R the cylinder radius); with increasing incident wave steepness of: $kH = 0.122$, 0.205, 0.286, and 0.385 respectively. The results compare well with the measurements and generally capture the high-order effects which are not part of second-order theory.

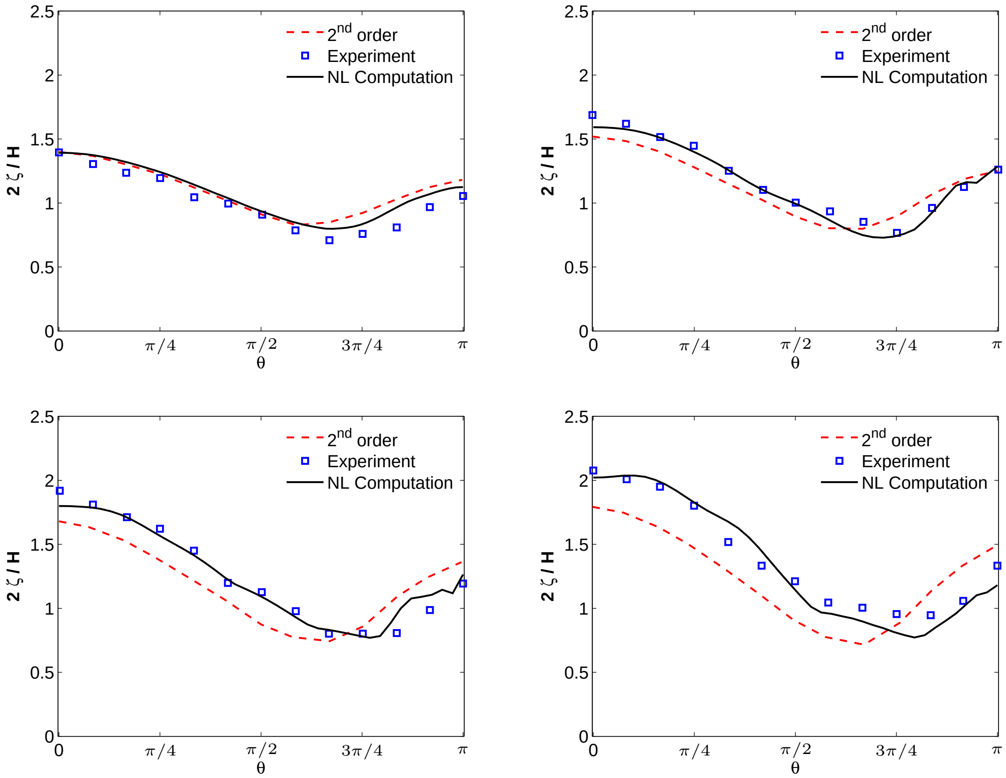

Figure 4: Nonlinear run-up around a bottom mounted circular cylinder compared with Stokes 2nd-order theory and experimental measurements by [5].

References

[1] H. B. Bingham and H. Zhang. On the accuracy of finite difference solutions for nonlinear water waves. *J. Engineering Math.*, 58:211–228, 2007.

[2] A. P. Engsig-Karup and H. B. Bingham. Boundary-fitted solutions for 3d nonlinear water wave-structure interaction. In 24^{th} *Intl. Wrkshp. Water Waves and Floating Bodies*, Zelenogorsk, 2009.

[3] A. P. Engsig-Karup, H. B. Bingham, and O. Lindberg. An efficient flexible-order model for 3D nonlinear water waves. *J. Comput. Phys.*, 228:2100–2118, 2009.

[4] J. D. Fenton. The numerical solution of steady water wave problems. *Comput. Geosci.*, 14(3):357–68, 1988.

[5] D. L. Kriebel. Nonlinear wave interaction with a vertical circular cylinder. Part II: wave run-up. *Ocean Engineering*, 19:75–99, 1992.

[6] R. Luquet, L Gentaz, P. Ferrant, and B. Alessandrini. Viscous flow simulation past a ship in waves using the SWENSE approach. In 25^{th} *Symp. on Naval Hydrodynamics*, St. John's, Newfoundland, 2004. ONR.

[7] G. Taylor. An experimental study of standing waves. *Proc. R. Soc. Lond. A*, 218:44–59, 1953.